Mathematics in Early Years Education

MEYE CURRICULAR DESIGN

Ana Ancheta Arrabal
(University of Valencia)
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Introduction:

The starting point of the project lies on the international comparison and documents (UNESCO, OECD, EU) about the suitability of an ECEC system to implement new curriculum reforms (Ancheta-Arrabal & Segura, 2022). The proposal is aimed at: educational services for all boys and girls from 1 year age (once parental leave ends); ALWAYS with A PEDAGOGICAL PAIR of professionals prepared for the education and care of young children and one of them AT LEAST is TRAINED SPECIFICALLY IN EARLY CHILDHOOD EDUCATION (higher education level); from a UNIFIED and CENTRALIZED CURRICULUM under the competence of the education ministry/authority; working in LEARNING COMMUNITIES that include families in the educational project of their children; with MATERIALS ADAPTED TO THE AGES of young children both in their nature and in their intentionality.

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The following document is structured around three blocks of content:

1.Foundation of the pedagogical and psychological basis that supports the proposal for the development of mathematical thinking in early childhood.

From a holistic and comprehensive approach of the right to early childhood education and care, as well as attending to educational processes in relation to socialization processes from the understanding of early childhood education services as learning settings, but also as spaces of and for life. Therefore, in the design of the proposal for each stage of this vital period, both cognitive development and social development are attended in order to understand that both are inseparable and indissoluble in child development.

2. MEYE curricular design. Proposal delimited by stages.

For each of the stages, we start from the universality of this language to the specificity of each life and educational trajectory. This means that even though the proposals are universal in nature, it is necessary to adapt the teaching-learning processes to the age, personality, and context of each student (Canals, 2018).

- Babies who can count (0-18 months old).
- Little explorers (18 months to 3 years of age).
- Playing specialists (3 years until entering the primary stage 7 years).

3. Central ideas for Mathematical Education in the Early Years (MEYE).

1 Foundation of the pedagogical and psychological foundations that supports the proposal for the development of mathematical thinking in early childhood:

The late increasing emphasis on young children's literacy and numeracy, frequently occurs at the expense of maintaining, let alone developing a broader pedagogical content knowledge for developing curriculum (Morton, McMenamin, Moore, & Molloy, 2012). The mathematical language has to be discovered by oneself to be understood, because it is like learning any other art as music is learned, it is necessary to attend to this complexity.

The formalization of the notions to be worked on during early childhood (such as those of natural numbers or topological geometry) is a part of the recent history of mathematical science. The complexity of mathematical notions can become a serious learning difficulty if the teacher does not develop didactics that take this fact into account. Hence, the mathematical language must be taught, and its learning depends on the age, personality and context of each student (Canals, 2018). Hence, the implementation of these guidelines will require careful consideration of how they fit in every national context and the application to every setting.

The analysis of different approaches and interpretations of the nature of mathematics allows us to formulate a conclusion concerning the grasp of the process of learning and teaching mathematics. Two approaches can be distinguished:

- Level of instrumental understanding (algorithmic): it assumes that teaching mathematics is based on skillful transfer of specific formulas, action schemes ready to use in practiced, typical situations/tasks.

- Level of relational understanding: assumes that activities that lead to discovering relationships, relations, mathematical structures and sometimes to their creation are important in learning mathematics, learning and understanding why procedures used function in this way (Polya, 1981, Freudenthal, 1986). Freudethal believes that mathematics is a social activity, an attitude, a way of mastering this world cognitively, practically and emotionally.

In the psychological concept of sociocultural constructivism, J. Bruner (1973, 1983) emphasizes the understanding of the learning process as cooperation, collaboration in the child-child and child-adult relationship. When performing tasks together, it is not only about meeting, but above all about exchanging meanings. In this way understanding is constructed together. Learning is a social activity because it ensures the negotiation of meanings and the construction of knowledge and skills in a specific context. Scaffolding in learning mathematics in Bruner's constructivist approach means creating for children to be active in three areas:

- **Enactive** ➡ representation of knowledge through action.

- **Iconic** ➡ visual representation.

- **Symbolic** ➡ use the words, numbers, music notation... to describe experiences.

The establishment of a zone of proximal development, according to Vygotsky's (1978) concept, in which the learning process can be scaffolded, is a key task for the teacher promoting children's activity. In addition to supporting the learning process, the educator is involved in co-creating meaning with the child, based on awareness and understanding. Vygotsky emphasizes the crucial importance of social interaction in the child's cognitive development and learning process. He places great emphasis on dialogue and interaction between the learner and other sand stressed that every developmental function, in thinking and language, appears first in a social relationship (interpsychological) and then individually (intrapsychological).

Boaler (2016) a mathematics didactician at Stanford University, works with psychologist C. Dweck and popularizes an approach to mathematics education that exposes the building of positive motivation. She emphasizes that mathematics in her interpretation is a science of relations, but it is also a form of communication. He believes that the basic reasons for failures in learning mathematics and the rapidly progressing process of students' demotivation stem from the fact that traditional education lacks engagement in thinking, speaking and the connection of analyzed issues with reality.

To the moment, it seems to be not sufficiently clarified whether language could be an influencing reason for differences of kindergarteners' mathematical performance from a cross-national perspective (Gasteiger, Brunner, & Chen, 2021: 115). Nevertheless, it is being assumed that children's participation in everyday cultural practices supported their mathematical communications (Worthington, Dobber, & van Oers, 2019: 94).

The question about the role of the cultural background and whether culture really matters is a broadly discussed and still open one (e.g., the influence of teachers' beliefs and their child-specific expectations which could be influenced by their culture in a similar way as parents' beliefs and attitudes are), but, besides language and culture, instruction and curriculum are mentioned as attempts to explain differences in children's outcomes (Gasteiger, Brunner, & Clean, 2021).

The results of research on the relationship between the level of development of language and mathematical skills in preschool children deserve attention because of their importance for educational practice. They indicate the association of the development of these two categories of skills. Detailed analyses show a relationship between narrative development in children aged 4-6 years and such categories of mathematical abilities like: numeracy skills (e.g. cardinality), math language (describing the relations), and patterning skills (discovering regularities) Cross-domain associations between mathematical and narrative abilities in preschool-aged children (Khan et al., 2021).

In an earlier study by J. Sarama et.al. (2012), a link was proven between early mathematics learning using the Building Blocks curriculum and oral language and literacy. The study was conducted in an experimental model with a control group. Preschool children in the experimental group developed the ability to recall key words, use of complex utterances, willingness to reproduce narratives independently, and inferential reasoning (practical content) at a significantly higher level than children in the control group.

PEDAGOGICAL GUIDELINES:

For young children, opportunities to learn arise from both naturally occurring, informal experiences and from planned activities. Typically occurring everyday activities provide the stimulus for much of children's informal mathematical development. Infants, for example, learn about time and pattern through the use of rhymes and song, and develop spatial skills and awareness as they move around their environment. Likewise, the everyday activities of sharing, cooking, playing games, completing puzzles, counting, estimating distances, and making music provide rich opportunities for young children to practice and develop mathematical competencies. Interacting with information and communication technologies (ICTs) provides a contemporary example of everyday experiences that can be linked to mathematical activity. (Clements, 2002; Anthony & Walsh, 2009).

Scaffolding and Vygotsky's concept of the ZPD. Scaffolding (i.e., assistance) is most effective:

- the support is matched to the needs of the learner.

- this puts them in a position to achieve success in an activity that they would previously not have been able to do alone.

Educator should:

- Tell the learners why they are learning.

- Provide opportunities for active engagement (cognitive, kinesthetic, social).

- Plan to use the learners' prior experiences.

- Plan to structure the learning experience based upon understanding of curriculum.

- Engage with the learners through dialogue and questioning.

- Be sensitive to the emotional aspects of learning experiences.

- Contextualize the activities with real-life examples.

Educators need to understand how mathematics learning is promoted by young children's engagement in play, and how best they can support that learning (Dooley et al., 2014).

On the context of play for the teaching and learning of mathematics in early childhood education, van Oers (2010) highlights the following two points:

- Children's learning should be embedded in a play-based curriculum, with teacher-led actions and imbued with mathematical meaning;

- In the role-play activities, children imitate cultural practices (like supermarket, restaurant, laboratory.), which form contexts for mathematical learning.

2 MEYE curricular design. Proposal delimited by stages.

The following guidelines are considered under the stages young children go through in the learning of mathematics. For each of them, we start from the universality of this language to the specificity of each life and educational pathway. This means that even though the proposals have a universal character, it is necessary to adapt the teaching-learning processes to the age, personality and context of each student (Canals, 2018). Therefore, we will develop this proposal under what we call mathematical pathways in early childhood education. The main objective is not that professionals have at their disposal an instrument to label children under 3 years old based on expected productions according to their chronological age, but rather that it can serve as a guide for professionals.

From a holistic and comprehensive approach to the right to early childhood education and care, as well as by attending to educational processes in relation to socialization processes from the understanding of children's education centers as learning spaces, but also as living spaces. Therefore, the design of the proposal for each stage of this vital period addresses both cognitive development and social development, understanding that both are inseparable and indissoluble in child development.

Babies who know how to count (0-18 months old).

According to De Castro, Flecha and Ramírez (2015), child math education has been progressively extended to younger and younger ages, including the period of 0 to 3 years (Fuson, Clements and Beckman, 2009; NAEYC and NCTM, 2013; NRC, 2014). As a result, the curriculums also begin to reflect the presence and relevance of child math activity from birth (de Castro, 2016; Fuson, Clements and Beckman, 2009). This widening of the age range means that early childhood education professionals need increasingly to accurately understand what intuitive and informal mathematical knowledge children under the age of 3 can begin to acquire, so some authors have begun to set age expectations (Geist, 2014). According to the European context and the comparative study the age at which young children can start attending an educational setting is around 12 months old. We consider this age as a possible start for informal mathematical knowledge that children under 3 years of age can learn, and more specifically, the range of actions they take during the process of acquiring the expected learning (Alsina&Roura, 2017). However, that allows to establish the levels of acquisition of informal mathematical knowledge before the age of 3 years, but does not set age expectations to make it easier for children's school professionals to interpret levels flexibly while preventing them from falling into the mistake of tagging children based on expected productions based on their chronological age.

According to Geist (2014):

- From 0 to 6 months: does not describe mathematical concepts.

- From 6 to 12 months: children of this age discover the permanence of objects (numerosity or cardinal number) and the valuation of distance (measurement).

- From 12- to 18 months: they internalize the concept "more", associated with the action of increasing the quantity of elements of a collection (numerosity); the measure of some elementary quantities such as length (measurement); the classifications (patterns, reasoning and algebra) and the matching by shape criteria (geometry and shapes).

Alsina and Berciano (2016) find that children under the age of 1 and a half basically recognize sensory characteristics of objects and compare (perceptually) collections based on the number of objects, while from the age of 2 they already begin to carry out actions more complex in relation to these content blocks (classifications, orderings, correspondences and series) and actions associated with the knowledge of positions, shapes and measurable attributes arise. To continue advancing in this direction and to be able to specify with greater precision the range of actions carried out by children under 3 years of age during the process of acquiring the knowledge described by Alsina (2015), in 2017 the construction and validation of the rubric "Acquisition of informal mathematical knowledge from 0 to 3 years" (ACMI 0-3)[3] was even presented with the purpose to provide guidelines to facilitate that Early childhood educators can flexibly interpret the levels of acquisition of mathematical knowledge of children under 3 years of age; rather than to establish expectations by age, as is the case of the contributions of Clements (2004) or Geist (2014), nor to analyze the frequency with which they carry out these actions (Alsina and Berciano, 2016; Alsina and León, 2016).

In sum, during this period it is important to take in account the Sensory Qualities learning (see Figure 3) that very young children can experience, and to plan every learning opportunity from the level children can be experiencing. Their body is the most important manipulative during this period (Søndergaard, 1996), but not representations as very young children are not able to use this thinking.

[3] For further information see: Alsina, A. y Roura, D. (2017). Estableciendo niveles de adquisición de conocimientos matemáticos informales antes de los 3 años: diseño, construcción y validación de una rúbrica. Edma 0-6: Educación Matemática en la Infancia, 6(1), 32-52 in http://www.edma0-6.es/index.php/edma0-6/article/view/24/21

▲ Little explorers (18 months of age up to 3 years).

Around the age of two, young children are beginning to explore their immediate and nearest environment. Therefore, it is important to create mathematics learning opportunities for them to explore and experience by dedicating specific time and integrate mathematics throughout the day. At this stage they develop their curiosity, progressively identify their needs, emotions and feelings, recognize themselves as a differentiated person and start in graphomotricity with their first doodle.

According to Geist (2014):

– From 18 to 24 months: sthe same concepts are included as in the 12 to 18 month range, plus multiple classification, that is, the possibility of making different classifications of the same collection of objects based on different criteria (patterns, reasoning and algebra).

– From 24 to 30 months: the children already make term-to-term correspondences, internalize the concept "one" and the ability counting (numbers); compare objects based on their size (measurement); perform sequences (patterns, reasoning, and algebra); and finally, they make stacks (geometry and shapes).

– From 30 to 36 months: the same items as in the 24 to 36 months range are included, plus the serializations.

In sum, the meaning is developed from the immediate environment of the student. Mathematical constructions must not be decontextualized to make sense. For the logical-mathematical learning in this first stage, special importance of the body as a means of communication with the world and of the game: symbolic, motor, with the body, with adapted material. Use of ICT, audiovisual media, the body and different symbolic systems to communicate ideas, patterns, etc. Groupings of objects, their spatial relationships, their properties and meaningful classifications that can be elaborated ...

■ Specialists in play (3 years until entry into primary stage 7 years).

– 3 years: at the age of three young children begin the creative stage, begin to show interest and skills to trim, follow lines, draw more precise shapes... They also begin to be placed in time, to assimilate habits and customs, and, very importantly, to communicate through the word. Children around this age also perfect graphomotricity with its first doodle, sharpens its ability to observe, its analysis of the environment and the different spaces.

– 4 years: at the age of four young children acquire a greater independence and autonomy.
They can classify objects and materials by colors, shapes, or numbers.
They have developed their spatial perception, and they are able to discern between inside and outside, up and down, or back and forth.
They might be able to use their imagination and fantasy in everyday situations. They like to participate in the responsibilities of the family/group and feel part of it.
They begin to expand their affective relationships, establishing ties with one or the other friends. It might be noticed that they like to tell and talk about their experiences, as well as they might be increasingly better understood.
In addition, they improve their ability to trim, follow lines, draw more precise shapes.

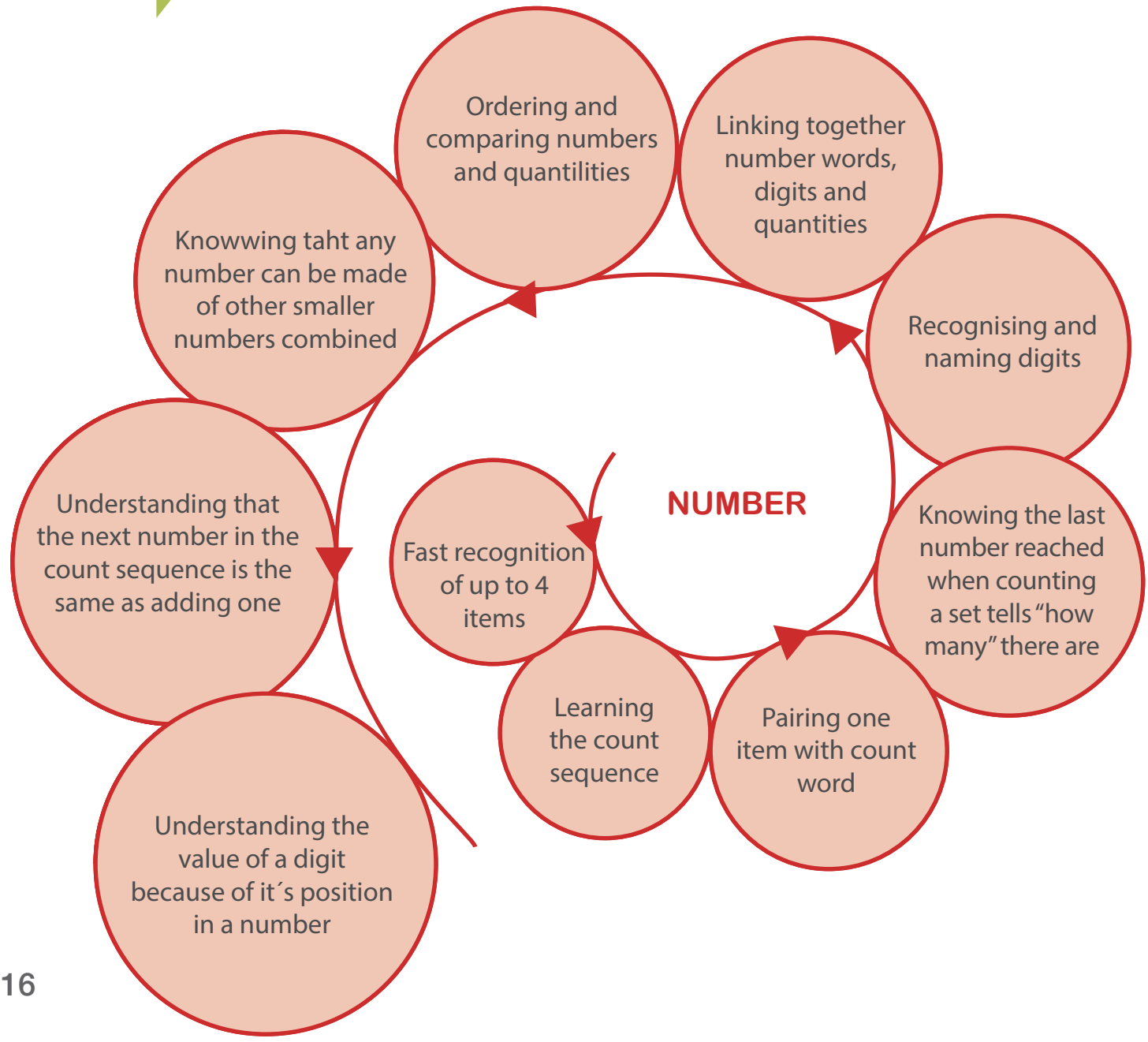
– 5 years: at the age of five young children have strengthened but also like their autonomy, they begin a more rapid learning as they are more intellectually mature and able to pay attention for longer periods of time.

At this age they have significantly developed language, as well as improved their mathematical logical learning (identify opposing concepts, reproduce geometric figures, classify objects, can count to ten…)

- Transitions to primary education (last year of early childhood education and first of primary education).

The following diagrams summarize what we know about number and operations development. The development of each skill or concept is not discrete, instead there is considerable overlap in development so children may develop several skills in parallel. Children may also move through the spiral in different orders. There is considerable overlap in development across these topics: e.g., understanding of operations builds upon children's understanding of numbers.

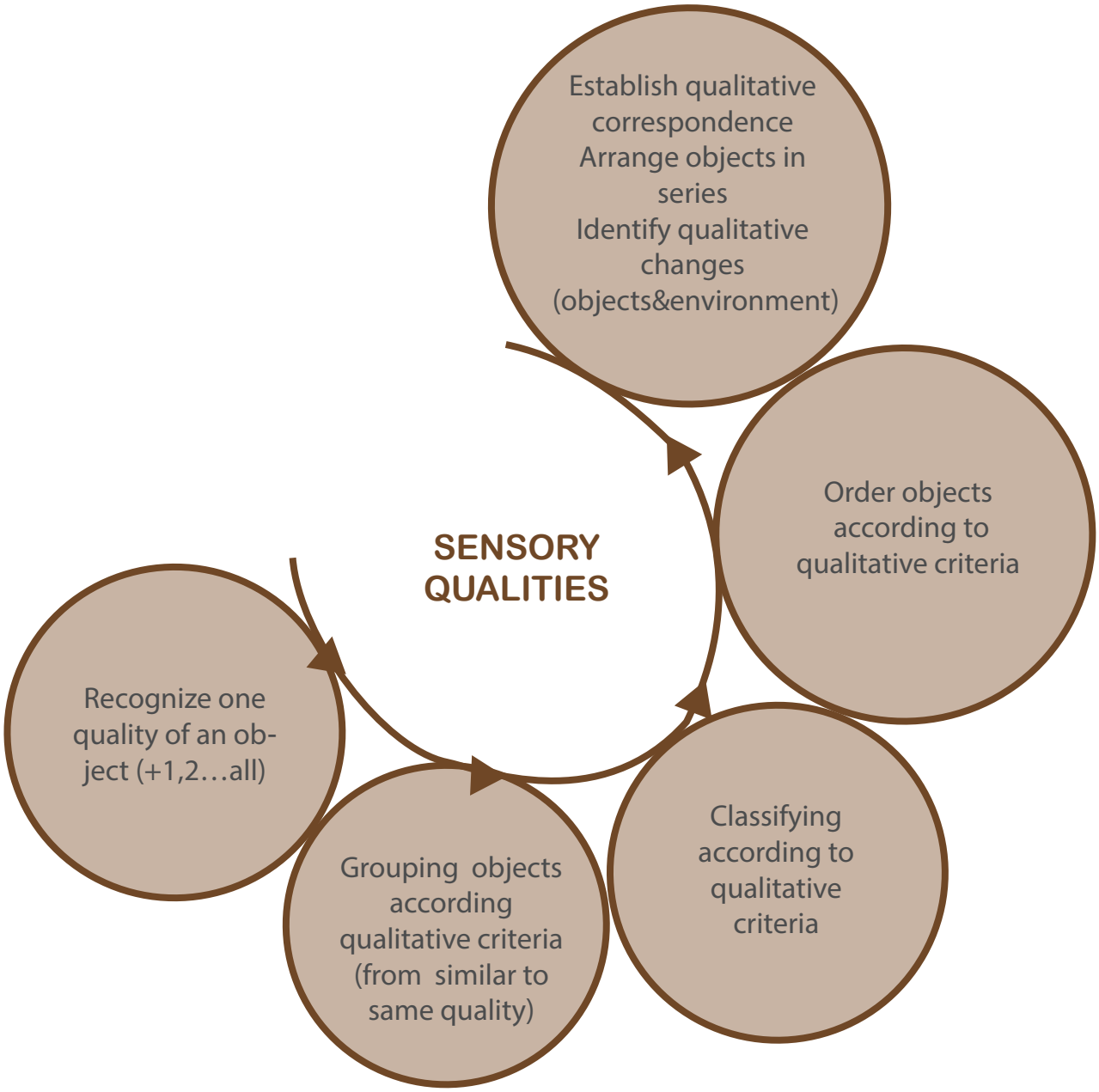
Figures 1 and 2.
Diagrams of mathematical learning development.



Source: Education Endowment Foundation (2020, p. 10).

To these diagrams, we can add according to Alsina&Roura (2017) another one around Sensory Qualities (Figure 3) which is part of the acquisition of informal mathematical knowledge. This progression in the learning is especially relevant at a very early age (between 0 and 3 year old) as a basis for the future learning of mathematics.

Figure 3.
Diagram of sensory qualities developments.



Source: Adapted from Alsina&Roura (2017).

3 Central ideas for Mathematical Education in the Early Years (MEYE).

The pedagogical principles from which to start in the learning of mathematics are meaningful learning, globalization, holistic aspects, manipulation, the relationship with nature and culture, activity, observation, routines and socialization. We have to start from equity and individualized attention. It is intended to achieve in the child to internalize, learn to learn, teach to think, to surprise, to start up their cognitive abilities. Arouse the interest of the child, presenting situations, activities that attract their interest and that they can relate to their previous experiences; to create these situations, so that learning is meaningful, intellectual, affective and social. At first, these activities are sensory and motor, they quickly become activity of thought, reflection and representation. With motor activities, reference is made to the body in action, the deep experience of a series of educational situations that will go to a higher plane, of a conceptual order, through symbols, images, plastic assemblies, structures figurative; this will be the means to activate your intellectual faculties. It must be a work of the senses and language, surprise, motivation and change, in a creative school in which the principles of freedom of action, knowing how to listen, participation, tolerance and autonomy, must be present. The motivation that in this stage has to go together with the activity, the search for something new, in the face of routine and monotony, is very important.

Introduction to Mathematical Thinking

· For the logical-mathematical learning in Early Years Education, special importance of the body as a means of communication with the world and of playing: symbolic, motor, with the body, with material.

· The meaning is elaborated from the student's immediate environment. Mathematical constructions must not be decontextualized, in order to make sense for the students.

· Groupings of objects, their spatial relations, their properties and the classifications with meaning that can be elaborated.

· The objectives of the Early Years Education curriculum in Spain affect:
play, student experience, contextualization, manipulative learning, relationships, classifications, order between elements of the immediate environment...

· Go from the simplest to the most complex.

· Task types:

a) Reproduction (already familiar contexts, already practiced knowledge, imitation, application).

b) Connection (less familiar contexts, managing and relating different systems of representation, problem solving strategies).

c) Reflection (creativity, use of concepts, generalizing and justifying results).

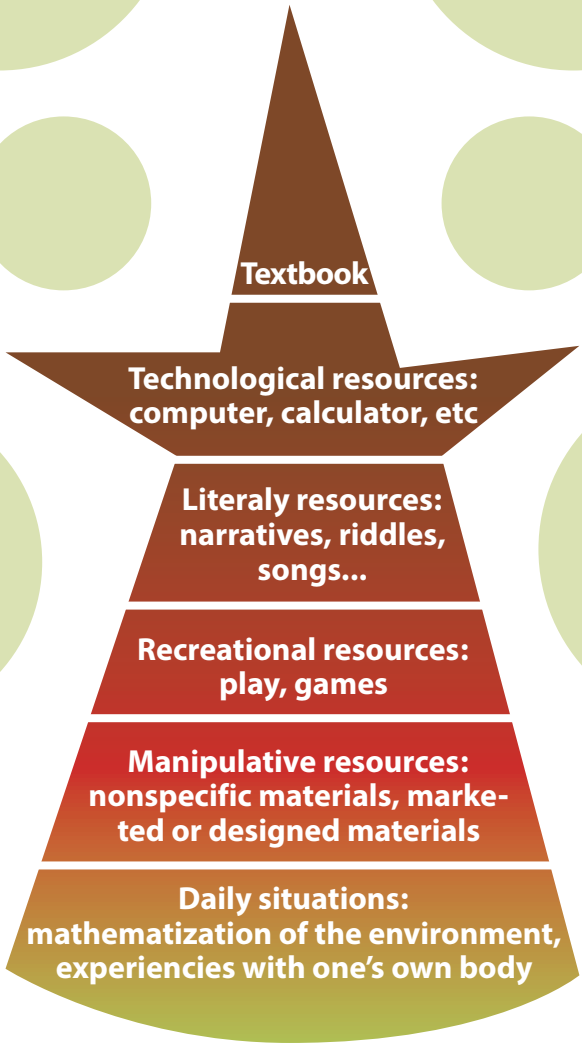
• Levels of development of mathematical thinking:

a) Concrete (manipulation, experience).

b) Graphic (graphic and symbolic representation).

c) Abstract (abstraction).

Figure 4. Pyramid of Mathematical Education.



Adapted from ALSINA, Àngel (2010, pp. 12-16).

Figure 4 illustrates the different levels in mathematical education in the early years which are described as:

· The first level is the basis of mathematical education, what must be worked on most with students: everyday situations, mathematization of the environment, experiences with one's own body.

· The second level is that of manipulative resources: nonspecific materials, marketed or designed.

· The third level is that of recreational resources: games.

· The fourth level is that of literary resources: narratives, riddles, songs.

· The fifth level is technological resources: computer, calculator.

· The last level, which corresponds to the formalization and systematization of learning, is the textbook.

Logical skills of students in Early Childhood Education:

· Classifications using logical criteria (quantitative or not):

· Logical thinking is acquired through inquiry, associated with the perception of the environment: to observe, compare, classify, order and series objects with respect to their properties. The two essential aspects of logical thinking at this stage are:

a) Classifications using logical criteria (quantitative or not):

- Are based on the idea of "equivalence relation", which means that all elements of the same class can be considered equivalent.
- Or in the idea of "order relation", by means of the position that some objects occupy with respect to others.

b) Series, patterns…Construction of a succession of elements (auditory, gestural, graphic...) following a rule. In rules of repetition, the different elements are presented periodically.

· Students develop skills in inference, predictions and deductions.

· Inquiry requires:

a) Identify characteristics that can be categorized
b) Ask questions.
c) Anticipate situations.
d) Prevent consequences.
e) Observe the effects of their actions.
f) Build relationships between observed phenomena.

· You can work with objects in the children's environment, asking them questions to discover their attributes. It is necessary to advance towards the comparison of attributes, which is the logical phase prior to quantification. Dynamize with questions: how are they similar? how are they different? which is bigger?

· Observation and comparison are the processes previous to making classifications and series.

Basic comparisons:
Long-Short / Big-small
High-low / Old-young
Fat-fine / Hot-cold
Wide-Stretch / Hard-soft
Fast-slow/ Lot-little
Close-far / All-none
First-last / Before-after

Figure 5 presents a summary of central ideas around the mathematical early years education according two early childhood cycles from 0 to 3 years old, and from 3 to 6 years old:

Figure 5.
Central ideas for Mathematical Education in the Early Years (MEYE):

MEYE CENTRAL IDEAS					
	Numbers and operations	**Relationships and change**	**Space, shape and measurement**	**Statistics and probability**	**Processes**
EI 0-3	We look at the amount.	To know, we compare and observe similarities and differences. This allows us to start organizing and to order.	Finding landmarks helps to position yourself and gives security.		Experimenting is essential and should be encouraged
EI 0-3	There are actions that change the amount.		Perception and experimentation lead to organizing space and time		Putting words into actions and discoveries promotes mental representation and thought
EI 0-3	Let's start saying numbers.		Objects have characteristics that make them predictable.		
EI 0-3	We begin to compare and assess length and weight.				
EI 3-6	Quantity does not have a single form.	To know, we compare and observe similarities and differences. That us allows you to organize and sort.	We can organize ourselves in both space and time and doing so helps us position ourselves.	To help us interpret phenomena and information we collect data.	There are situations that lead us to ask ourselves questions.
EI 3-6	We can know in a group, "how many elements are there" at a glance or counting.		Real objects resemble geometric figures.	There are things that are safe, others impossible, and some that are likely.	Representing helps us understand and find ways to resolve.
EI 3-6	The numbers have an order.		We can compare and measure length, area, volume, weight, capacity, time…		Talking helps to order thinking and sharing.
EI 3-6	Numbers can be composed and decomposed.				We need to be aware of the reasons that lead us to think something and check them out.
EI 3-6	There are specific situations that lead us to add and subtract.				

Source: Adapted from Torra & Roca (2019).

Teaching-learning of mathematical aspects:

The formation of the mathematical field in early childhood education from 0 to 8 years old, can be considered as basic for later learning that is based on these first ways of understanding the world by the young children. For this, it is necessary to establish several didactic and educational intervention areas:

- From the routines of the classroom and the use of the assembly, the day to day becomes an educational factor in the fundamental mathematical field: what day, month and year we are, in what season of the year and what weather is the weather (link with nature), how many boys and girls have come to class, how many have not come, the class number that each child has assigned ... Personal changes, the child's growth, the child's birthday and their relatives and acquaintances.

- From the culture, the traditional game, the educational setting opens up to the context and thus, mathematics is part of the life of the classroom and the basic concepts of this stage are learned from inductive processes (spatial, temporal, measurement notions ...)

- Starting from the relationship between the areas, through globalizing and significant activities.

- The Projects, as the centers of interest, are part of those themes that encompass the life of the classroom, which the boys and girls decide and involve families to get to know the world and relate all the learning.

- From a globalized and intercultural vision of the world and from inclusion, we can understand the world that surrounds us with a vision of solidarity and responsibility and bring this look closer to the youngest children to also bring it closer to families. Families in this stage have a huge collaborative and implication potential, they have to suppose a great support, understanding the dissemination of education at all levels, in which the nursery school or the school collaborate with the collective and society education.

- Already specifically, training in specific mathematical concepts through workshops, corners, specific games ... and specifically in the last two courses of the stage, the four and five years ... such as the Montessori methodology, the specific materials (Cuisenaire strips, abacus, logic blocks ...),

- In learning mathematics, the basic activities of classifying, matching, ordering (most minor ...), developing series, adding and subtracting ... must be present, with their different levels of complexity.

To conclude, it is essential to reflect these aspects in a document, but it is more important to observe how the learning of mathematics unfolds on a daily basis in early childhood education settings. It is noteworthy the task of education professionals in this educational stage, their dedication, motivation, capacity for innovation and to reach every boy and girl; collaboration with families and teamwork. This very basic stage and at the same time with its own entity, should have other working and organizational conditions, and greater social recognition. Mathematical language must be taught and its learning depends on the age, personality and context of each student (Canals, 2018). At last, some of the frequent mistakes to avoid are presented in the following decalogue to consider when working mathematics in early years:

Decalogue of errors to avoid working mathematics with young children (3-6 years) (Maria Antònia Canals, 2018):

1. Show large, highly visible written numbers in a fixed and persistent way in the classroom.

2. Reduce the count to just one memory job, with the sole purpose of remembering the name of some spellings.

3. Identify the Dienes logical blocks with the geometric names of their shapes.

4. Represent the first sums with small balls (necklace type) without taking into account the space occupied by the wire that connects them at the ends of each quantity.

5. Introduce the use of plain colored strips for understanding natural numbers before reaching the first cycle of primary simply because they are very beautiful.

6. Neglecting the in-depth treatment of basic activities in mathematics: notion of quantity and changes; movements in space; lines, surfaces and bodies; and solving real situations, which are the real problems.

7. Use the materials that are beautiful and expensive without seeing the ones that really have a more authentic relationship with the life of the class and the teachers consider appropriate for each child, especially those who have difficulties.

8. Forgetting to complete all the activities with the children's verbal expression of what has been done, what has happened, what we have discovered, etc.

9. Being victims of a program that we had drawn up or imposed on us.10. Not enough to confront our work, successes and difficulties with our team of work.

10. Not confronting our work, successes and difficulties with our work team.

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